UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS

MODULE: **ASMA2B4**

CAMPUS: **APK**

EXAM: **JUNE 2014**

DATE:	**14 JUNE 2014**	**SESSION: 12:30-14:30**
ASSESSOR(S):	**C MARAIS**	
INTERNAL MODERATOR:	**Y JACOBS**	
DURATION:	**2 HOURS**	**MARKS: 50**

SURNAME AND INITIALS:

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STUDENT NUMBER:

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CONTACT NR:

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NUMBER OF PAGES: 10 PAGES

INSTRUCTIONS: **ANSWER ALL THE QUESTIONS IN PEN**
YOU MAY USE A CALCULATOR

<u>Question 1</u>

Answer the following True or False questions and give a short justification/counter-example:

a) The set of odd integers under addition form a group. [2]

TRUE	
FALSE	

b) Let G be a group and let $a, b \in G$. Then $(ab)^{-1} = a^{-1}b^{-1}$. [2]

TRUE	
FALSE	

c) A cyclic group has a unique generator. [2]

TRUE	
FALSE	

d) Let H be a subgroup of G and let $a,b \in G$. If $Ha = Hb$ then $b \in Ha$. [2]

TRUE	
FALSE	

e) The relation of being isomorphic is an equivalence relation on the collection of all groups. [2]

TRUE	
FALSE	

f) If φ is an onto homomorphism from a group G_1 to a group G_2, then $G_1 / Ker\varphi \approx G_2$. [2]

TRUE	
FALSE	

Question 2

a) Show that $\mathbb{Z}_{11}$ is cyclic by finding a generator for it. [1]

b) Are there any non-trivial subgroups for $\mathbb{Z}_{11}$? If so, give an example of such a subgroup, and if not, explain why it is not possible. [2]

Question 3

Consider the group S_7 of all permutations of the set $\{1,2,3,4,5,6,7\}$. Let $\alpha=(1,7,2,5,4)(1,4,2,3)(1,5,4,6,3,2)$ and $\beta=(1,3,5,7)(2,4,6)$ be permutations in S_7.

a) Write α as a product of disjoint cycles. [1]

b) What is the order of β? [1]

c) Write $\alpha\beta$ as a product of disjoint cycles. [1]

Question 4

Show that $U(8)$ and $U(10)$ are not isomorphic. [2]

Question 5

Let $G=\{(1),(132)(465)(78),(132)(465),(123)(456),(123)(456)(78),(78)\}$.

a) Find the orbit of 1. [1]

b) Find the stabiliser of 7. [1]

<u>Question 6</u>

a) What is the order of the group $\mathbb{Z}_3 \oplus \mathbb{Z}_5$? [1]

b) Show that $\mathbb{Z}_3 \oplus \mathbb{Z}_5$ is cyclic and find a generator for this group. [2]

c) In $\mathbb{Z}_3 \oplus \mathbb{Z}_5$ find a subgroup of order 3. [2]

d) In $\mathbb{Z}_3 \oplus \mathbb{Z}_5$ find a subgroup isomorphic to $\mathbb{Z}_5$. [2]

Question 7

If H is a subgroup of G and K is a normal subgroup of G, prove that $H \cap K$ is a normal subgroup of H. (Recall: $H \cap K = \{x \in G \mid x \in H \text{ and } x \in K\}$) Hint: First show that $H \cap K$ is a subgroup of H and then show that it is normal. [6]

<u>Question 8</u>

Let $G = \mathbb{Z}_{12}$ and $H = \langle 4 \rangle$.

a) Find the left cosets of H in G. [1]

b) Why can we say that $H \lhd G$? [1]

c) Construct a Cayley Table for the Factor Group $\mathbb{Z}_{12} / \langle 4 \rangle$ and show that it is cyclic. Hence find an n such that $\mathbb{Z}_{12} / \langle 4 \rangle \approx \mathbb{Z}_n$. [4]

Question 9

a) Give the definition of a Group Homomorphism. [2]

b) Let φ be a group homomorphism from a group G_1 to a group G_2. Prove that $Ker\varphi$ is a normal subgroup of G_1. [5]

Question 10

Let G be a group and let $\varphi : G \to G$ be defined by $\varphi(g) = g^{-1}$. If φ is a homomorphism, prove that G is Abelian. Hint you may use the fact that G is Abelian if and only if $(g_1 g_2)^{-1} = g_1^{-1} g_2^{-1}$ for all $g_1, g_2 \in G$. [3]